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Thermally stressed vibration of composite plates and shells

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Abstract— **In this research article, the characteristics of vibration of thermally stressed composite laminated shells are considered. Since, aerospace structures like space craft and defence structures like missiles are subjected to aerodynamic heating and they are most susceptible to vibration. Finite element method of analysis using semiloof shell element is employed in the formulation of governing equations. The thermal load can be specified by means of nodal temperature, varying over the surface as well as through the thickness. The governing equation includes the mass matrix, the geometric stiffness matrix and structural stiffness matrix. The validation checks on the program are carried out using results on homogeneous isotropic and laminated composite structures are available in the literature. Parametric studies are carried out for: cross ply and angle play laminations with symmetry and anti symmetry stacking sequence; simply supported, clamped boundary conditions; varying ratio of coefficient of linear thermal expansion; Varying number of layers for given laminated thickness; thickness to radius ratio.**

***Keywords*— Composite; Semiloof; Thermally stressed; Shell; Plate; Laminate; Vibration**

I. Introduction

The study on the dynamic behaviour of a structure is a part of structural analysis, and this assessment gives data which would aid in proficient design. This work involves studies on vibrational characteristics of laminated composite shells in the presence of thermal stress (initial stress) using finite element method of solution. The formulation used here is applicable for both plate and shell elements. In the recent past, several structural components are made up of advanced composite materials, which have been extensively used in aerospace structural components [1]. The vibrational characteristics are affected in the presence of thermal stresses, which may be developed in the aerospace structural components. The frequencies will vary depending upon the nature of stress. In a practical situation, the initial stresses developed may be predominantly of compressive nature due to thermal effects or inertia forces. It is evident that the frequency decreases as the initial stress increases.

The vibration analysis of structural members (beam, plate, shell, etc.,) plays a vital role in applications of mechanical, aerospace, and civil engineering [2]. Thermally induced vibration is associated with elevated temperatures and rapid heating [3]. A change in temperature of a body causes a change in its dimension, proportional to the change in temperature and its initial dimensions. Thus, thermal strain will develop in the body as a result of temperature changes. The thermal strain, e_T is equal to the product of a co-efficient of thermal expansion α, of the body and the change in temperature ΔT. However, in the case of an orthotropic material such as a unidirectional lamina, the coefficient of thermal expansion changes with direction. Thus, temperature change ΔT results in unequal thermal strains in the longitudinal as well as transverse directions given by the following equation:

$$e_L^T = \alpha_L \Delta T \quad \text{and} \quad e_T^T = \alpha_T \Delta T \qquad (1)$$

where, α_L and α_T are coefficients of thermal expansion is the longitudinal and transverse directions, respectively.

The vibration characteristics of shells are affected by various factors under practical situations, these factors includes: initial stress, anisotropy, heterogeneity, shear deformation, large deflection, surrounding medium and, variable thickness. Comparing plate and shell vibration, it is found that shell frequencies are more closely spaced and less easily identifies both analytically, and experimentally. Some of these factors cause decrease in frequencies and some cause increase in frequencies. The relationship between the square of frequency and these factors may be linear, piecewise linear or non-linear depending upon the material, boundary conditions etc.

In relatively recent research studies, the consequence of heat on the bending analysis of cross-ply square plates have been carried out [4], moreover, small amplitude vibration characteristics of thermally stressed laminated composite skew plates have been assessed [5]. Also, linear buckling analysis of laminated composite conical and cylindrical shells under thermal load has been studied in previous research works [6]. Dhanaraj and Palaninathan [7] used semiloof shell element formulation to obtain the fundamental frequency of laminated composite plate under initial stress. Ram et al., [8] investigated the effects of temperature on the free vibration of laminated composite plates using the finite element method. And, the final results were presented showing the drop in the natural frequency with the increase in uniform temperature for different laminate with varying boundary conditions.

In this research study, the effect of thermal loads, due to uniform temperature, on the vibrational characteristics of angle-ply and cross-ply laminated with different support conditions have been studied. This is a novel work, as a research study similar to this has not been conducted in the past.

II. Finite Element Formulation

In this research work the semiloof shell element developed by Irons [9] has been used. The versatility and good performance of this element has been demonstrated well as seen in literatures [6], [7]. For the analysis of the general class of laminates, shell elements which exhibit coupled actions of both bending and membrane behaviours are appropriate. This also avoids the use of two individual elements for the initial stress calculation and for the vibration analysis. The governing equations for the element considered are derived from the Lagrangian equations.

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} = 0, \quad i = 0,1,2,\ldots.n \qquad (2)$$

where, n is the degrees of freedom, and L is the Lagrangian, defined as:

$L = T - \pi$

where, T is Kinetic energy, and π is total Potential energy.

The kinetic energy (3) and total potential energy equation (4) are as follows:

$$T = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} \qquad (3)$$

$$\pi = \sum \int \frac{1}{2}\left[\{q\}^T[H]^T\begin{bmatrix} A & B \\ B & D \end{bmatrix}[H]\{q\} + \{q\}^T[G]^T[P][G]\{q\}\right] da \qquad (4)$$

The kinetic energy equation (3), and total potential energy equation (4) are substituted in Lagrangian equation (2), to give the following equation:

$$L = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} - \frac{1}{2}\{q\}^T\left[[K_S] + [K_G]\right]\{q\} \quad (5)$$

By substituting the above equation (5) for L in equation (2), then the governing equation can be written as,

$$\left[[K_S] + [K_G]\right]\{q\} + [M]\{\ddot{q}\} = 0 \qquad (6)$$

The governing equation for the vibration of laminated shells in the presence of thermal stress can be expressed as,

$$\left[[K_S] + [K_G]\right]\{q\} + [M]\{\ddot{q}\} = 0 \qquad (7)$$

By assuming that the vibration is harmonic, equation (7) can be deduced to equation (8).

$$\left[[K_S] + [K_G] - \omega^2[M]\right]\{q\} = 0 \qquad (8)$$

where,

$[K_S]$ is the structural stiffness matrix
$[K_G]$ is the geometric stiffness matrix
$[M]$ is the mass matrix
{q} is the vector of nodal degrees-of-freedom
ω is the angular frequency

By carrying out the Eigen analysis, the frequency of vibration and associated mode shape are obtained.

The strain at any point in a laminated plate and shell may be written in terms of the reference strain 'e' and change of curvature 'κ'. The components in the strain vector {e} are defined as:

$$\bar{e}_{xx} = e_{xx} + zk_{xx}$$
$$\bar{e}_{yy} = e_{yy} + zk_{yy}$$
$$\bar{e}_{xy} = e_{xy} + zk_{xy}$$

where,

$$e_{xx} = u_x$$
$$e_{yy} = v_y$$
$$e_{xy} = u_y + v_x$$
$$k_{xx} = w_{xx}$$
$$k_{yy} = w_{yy}$$
$$k_{xy} = 2w_{xy}$$

We can write,

$$e = \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix} \text{ and } [k] = \begin{bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{bmatrix}$$

The strain energy of the element,

$$\int \frac{1}{2} e^T.\sigma.dv = \int \frac{1}{2}([e] + z[k])^T[\bar{Q}]\,[e - e_T]dv$$
$$\int \frac{1}{2} e^T.\sigma.dv = \int \frac{1}{2}([e] + z[k])^T[\bar{Q}]\,[e + z(k) - e_T]dv$$
$$\int \frac{1}{2} e^T.\sigma.dv = \int \frac{1}{2}([e] + z[k])^T[\bar{Q}]\,[e + z(k)]dv - \int \frac{1}{2}([e] + z[k])^T[\bar{Q}]\,[\alpha]T.dv$$

The principle of minimization of total potential energy is used in deriving the governing equations. The total potential for the laminated structure subjected to thermal load may be written as:

$$\pi = \sum^{m} \int \frac{1}{2}\begin{bmatrix} e \\ k \end{bmatrix}^T \begin{Bmatrix} N \\ M \end{Bmatrix}\begin{bmatrix} e \\ k \end{bmatrix} da - \int \frac{1}{2}\begin{bmatrix} e \\ k \end{bmatrix}^T \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} da$$

Where,

m is the number of elements
a is the area of element
N is the stress resultant
M is the moment resultant
N^T is the thermal stress resultant
M^T is the thermal moment resultant

These stress and moment resultants are defined as follows with respect to:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{Bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{Bmatrix}\begin{Bmatrix} e \\ k \end{Bmatrix}$$

The A, B, and D matrices are called as extensional stiffness, coupling stiffness, and bending stiffness, respectively.

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \sum_{k=1}^{nl} \int_{h_{k-1}}^{h_k} \left[\bar{Q}_{ij}\right](1, z, z^2)dz$$

where,

$\left[\bar{Q}_{ij}\right]$ is the transformed reduced stiffness matrix nl is the number of layers

$$(N_{ij}^T, M_{ij}^T) = \sum_{k=1}^{nl} \int_{h_{k-1}}^{h_k} \left[\overline{Q}_{ij}\right] [\alpha]_j T(1,z) dz$$

where T is the temperature rise.

$$[\propto] = \begin{bmatrix} \propto_x \\ \propto_y \\ 2\propto_{xy} \end{bmatrix}$$

[α] is the vector of coefficients of thermal expansion.

III. Results and Discussion

The effect of temperature on the structural behaviour plays an important role in the analysis and design of structural components, particularly, like those used in aircraft structures, reactor vessels, etc., are vulnerable to thermal loadings. This study has been carried out for the case of laminates subjected to uniform temperature through the surface. The results presented here are based on the assumption that the material properties are implicit. It is to be noted here that unlike the mechanical loading, for a given temperature condition, the initial stress developed will depend very much upon the material properties in the present study. The material properties considered are, E_{ll}/E_{tt} = 10, G_{tt}/E_{tt}= 0.5, and μ_{lt} = 0.25.

The variation of frequency of orthotropic square plates with simply supported boundary conditions. The plate is subjected to constant and linearly varying temperature over the surface. Results are obtained using the present formulation is compared with Dhanaraj [7]. And, the results are shown in Fig. 1, and Fig. 2.

Simply supported S1: u≠0, v=0, w=0, θ≠0, for x=0,a
u=0, v≠0, w=0, θ≠0, for y=0,b

Thermal loading co-efficient, $\tau = \alpha_t T(a/h)^2$

T – temperature in ˚C

The material and dimensional properties of both antisymmetric and symmetric angle ply laminates used are as follows:

E_{ll}/E_{tt} = 15, G_{lt}/E_{tt}= 0.5, μ = 0.25, α_{tt}/α_{ll} = 2, a/h = 100, a/b = 1.

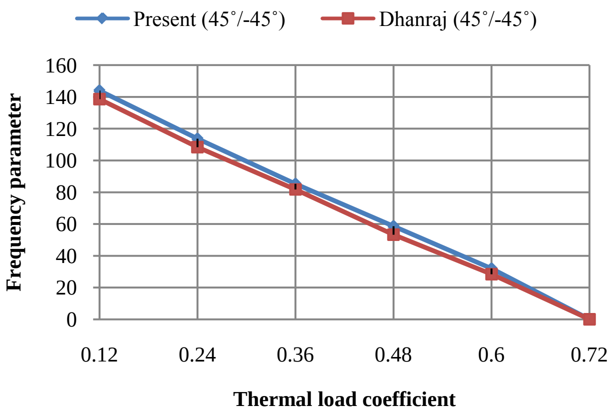

Fig. 1 Thermal load coefficient ($\tau = \alpha_t T(a/h)^2$) versus Frequency parameter ($\Omega = \omega^2 \rho b^4/E_t h^2$) for anti-symmetric angle ply laminates

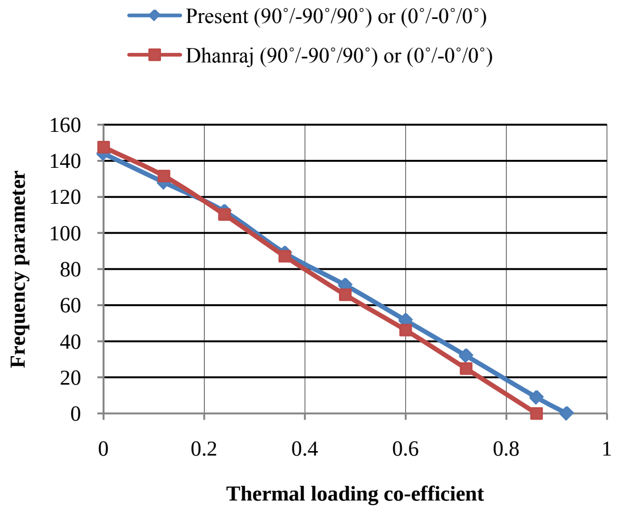

Fig. 2 Thermal load coefficient ($\tau = \alpha_t T(a/h)^2$) versus Frequency parameter ($\Omega = \omega^2 \rho b^4/E_t h^2$) for symmetric angle ply laminates

An isotropic homogeneous square plate (a/h = 100) is analysed. Using the simply supported conditions with aspect ratio α = 1, and α = 2. The exact solution of this problem is available in Ref. [7], and [10]. A quarter plate is modelled using a 4×4 mesh. The dimensional and material properties Comparisons of the exact values with present results are shown in Table 1.

TABLE I

Frequency Parameter for an Isotropic Plate

α	Simply supported			Clamped		
	Leissa	Dhanaraj	Present	Leissa	Dhanaraj	Present
1	19.74	19.72	19.17	35.99	35.93	34.89
2	49.35	49.28	49.92	98.59	98.29	95.12

A. *Angle-ply laminate*

The variation of frequency parameter (Ω) with the thermal load for symmetric angle-ply laminates are presented in the following two boundary conditions: simply supported edge, clamped edges. The variation of fundamental frequency parameter with uniform temperature rise for a symmetric angle-ply cross-ply laminates (15˚/-15˚/15˚) under simply supported conditions (u=0, v≠0, w=0, θ≠0) are shown in Fig. 3. The nature of variation is found to be fully linear or piecewise linear for all fibre orientation angles. The effect of temperature on the frequency of vibration a symmetric angle-ply laminate (15˚/-15˚/15˚) under clamped conditions is shown in Fig. 3.

B. *Cross-ply laminate*

The effect of temperature on the vibrational characteristics for cross-ply laminates with simply supported and clamped support conditions is considered in this section. The variation of frequency parameter with the temperature for a symmetric laminates under simply supported conditions are shown in Fig.

3. The nature of variation is found to be fully linear or piecewise linear. As observed in the case of cross-ply laminates, here also the temperature values are higher for cross-ply laminates compared to angle ply laminates for all supported conditions. The variation of frequency parameter with temperature for symmetric (0˚/90˚/0˚) and anti-symmetric (0˚/90˚) are shown in Fig. 3 and Fig 4, respectively. The nature of the variation found to be piecewise linear. Compare anti-symmetric laminates having maximum range of frequency values for symmetric laminates.

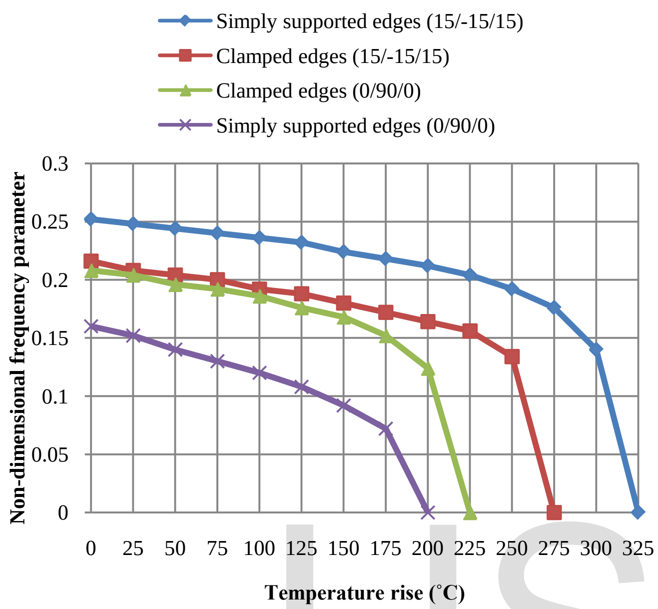

Fig. 3 Temperature rise versus non-dimensional frequency parameter ($\Omega=\omega R(\rho/E_{tt})^{1/2}$) for various cross-ply and angle ply symmetric laminates

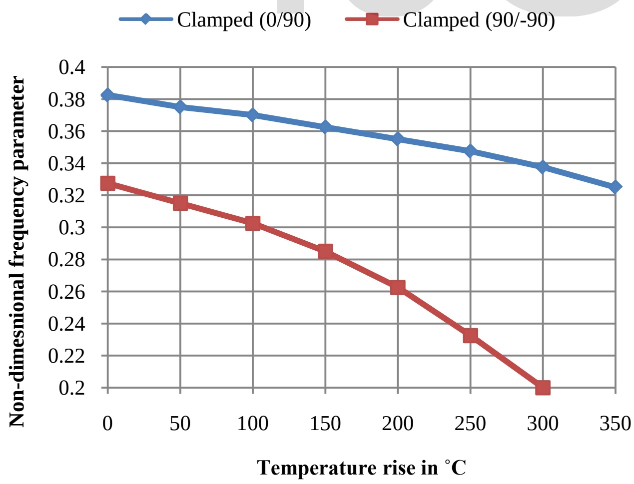

Fig. 4 Temperature rise versus non-dimensional frequency parameter ($\Omega=\omega R(\rho/E_{tt})^{1/2}$) for various cross-ply and angle ply antisymmetric laminates

The variations of frequency with respect to fibre orientation angle for a two and three layer simply supported laminates. The uniform for 0˚, 90˚ fibre orientation even though number of layer increases the same frequency occurs. It predicts same frequency hence there is no effect on frequency by increasing the layers. Other fibre orientation angle gives different frequencies. Also observed when fibre orientation increases from 0˚ to 90˚ the frequency values come down and reaches a minimum at 90˚.

IV. Conclusions

The effect of thermal loads, due to uniform temperature, on the vibrational characteristics of angle-ply and cross-ply laminates with different support conditions have been considered in this research article. The following are the conclusions drawn from the results presented here:

1. It can be seen that for all types of laminates and support conditions considered the nature of variation of the frequency with thermal load is found to be linear or piecewise linear.
2. The natural frequency of dependant on the boundary condition, especially in the case of laminated plates.
3. The frequency range is highest for clamped edge, and lower for simply supported edge condition.
4. Also, when the thickness to radius ratio increases the frequency value decreases in laminated plates.

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